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Conservation laws and tachyon potentials in the sliver frame

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Abstract

Conservation laws have provided an elegant and efficient tool to evaluate the open string field theory interaction vertex, they have been originally implemented in the case where the string field is expanded in the Virasoro basis. In this work we derive conservation laws in the case where the string field is expanded in the so-called sliver \mathcal{L}_0 -basis. As an application of these conservation laws derived in the sliver frame, we compute the open string field action relevant to the tachyon condensation and in order to present not only an illustration but also an additional information, we evaluate the action without imposing a gauge choice.

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1 Introduction

Since Schnabl’s discovery of the first analytic solution for tachyon condensation [1], in the last few years string field theory has come to the surface as a suitable framework to verify analytically Sen’s conjectures [2, 3]. Schnabl’s solution successfully proves Sen’s first conjecture which states that at the stationary point of the tachyon potential on a D25-brane of open bosonic string theory, the negative energy density exactly cancels the tension of the D25-brane. The second conjecture states that there are lump solutions which describe lower dimensional D-branes; and the third conjecture tell us that in the nonperturbative tachyon vacuum all open string degrees of freedom must disappear¹. Witten’s formulation of open bosonic string field theory [11] has been proven to be the suitable framework to test Sen’s conjectures [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45].

Historically the tachyonic instability in Witten’s cubic open string field theory has been analyzed by looking at the extremum of the tachyon potential at which the total negative potential energy exactly cancels the tension of the D25-brane. The tachyon potential has been computed numerically using the level truncation scheme [12, 13, 14, 15, 16, 17, 18]. This scheme originally introduced by Kosteletzky and Samuel is based on the realization that by truncating the string field to its low lying modes, namely keeping only Fock states

¹There are similar conjectures for the open string tachyon on a non-BPS D-brane and the tachyon living on the brane-antibrane pair [4, 5, 6, 7, 8, 9, 10].

with L_0 eigenvalue less than h , one obtains an approximation that gets more accurate as the level h is increased. Therefore in order to perform level truncation computations, the string field was traditionally expanded in the usual Virasoro basis of L_0 eigenstates. However, it is well known that in this basis calculations involving the cubic interaction term becomes cumbersome and the three-string vertex that defines the star product in the string field algebra is complicated [46, 47, 48, 49]. We can overcome these technical issues (related to the definition of the star product) by using a new coordinate system [1].

Let us recall that the open string worldsheet is usually parameterized by a complex strip coordinate $w = \sigma + i\tau$, $\sigma \in [0, \pi]$, or by $z = -e^{-iw}$, which takes values on the upper half plane. As shown in [1], the gluing conditions entering into the geometrical definition of the star product simplify if one uses another coordinate system, $\tilde{z} = \arctan z$, in which the upper half plane looks as a semi-infinite cylinder of circumference π . In this new coordinate system, which we refer as the *sliver frame*, it is possible to write down simple, closed expressions for arbitrary star products within the subalgebra generated by Fock space states. Elements of this subalgebra are known in the literature [50, 51, 52] as wedge states with insertions.

Using this new coordinate system, Schnabl's analytic solution of the open bosonic string field theory equations of motion was found by expanding the string field in a basis of \mathcal{L}_0 eigenstates [1]. The operator \mathcal{L}_0 is the zero mode of the worldsheet energy momentum tensor in the sliver frame. By a conformal transformation it can be written as

$$\mathcal{L}_0 = \oint \frac{dz}{2\pi i} (1 + z^2) \arctan z T(z) = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2 - 1} L_{2k} , \quad (1.1)$$

where the L_n 's are the ordinary Virasoro generators with zero central charge $c = 0$ of the total matter and ghost conformal field theory.

Since in open bosonic string field theory, the basic field is an object of ghost number one, a rather generic string field Ψ can be written as

$$\Psi = \sum_{n,p} f_{n,p} \hat{\mathcal{L}}^n \tilde{c}_p |0\rangle + \sum_{n,p,q} f_{n,p,q} \hat{\mathcal{B}} \hat{\mathcal{L}}^n \tilde{c}_p \tilde{c}_q |0\rangle , \quad (1.2)$$

where $n = 0, 1, 2, \dots$, and $p, q = 1, 0, -1, -2, \dots$. The operators $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$ and \tilde{c}_p are defined in the sliver frame [1], and they are related to the worldsheet energy momentum tensor, the b and c ghosts fields respectively

$$\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger = \oint \frac{dz}{2\pi i} (1 + z^2) (\arctan z + \operatorname{arccot} z) T(z) , \quad (1.3)$$

$$\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger = \oint \frac{dz}{2\pi i} (1 + z^2) (\arctan z + \operatorname{arccot} z) b(z) , \quad (1.4)$$

$$\tilde{c}_p = \oint \frac{dz}{2\pi i} \frac{1}{(1 + z^2)^2} (\arctan z)^{p-2} c(z) . \quad (1.5)$$

To find an analytic solution which represents to the tachyon vacuum, we impose some gauge and plug the string field (1.2) into the equation of motion $Q_B\Psi + \Psi * \Psi = 0$ so that the coefficients $f_{n,p}$ and $f_{n,p,q}$ can be determined level by level. For example, imposing the gauge $\mathcal{B}_0\Psi = 0$, Schnabl was able to determine these coefficients in terms of the celebrated Bernoulli numbers [1]. Once we have an analytic solution, in order to compute the value of the vacuum energy, we replace the solution Ψ into the string field theory action

$$S = -\left[\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right]. \quad (1.6)$$

At this stage it is clear that to evaluate the vacuum energy, we will need to compute correlation functions involving the two and three point interaction vertex. For vertex operators given in terms of the basic $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$, and \tilde{c}_p operators, to compute these correlation functions, straightforward techniques have been developed. As it is shown in [27], correlation functions involving the two point vertex, namely the BPZ inner product, can be expressed in terms of a known function \mathcal{F} evaluated at some particular points. On the other hand correlators involving the three point interaction vertex, namely correlators involving the $*$ -product can be expressed in terms of triple contour integrals having as an integrand the function \mathcal{F} . In reference [27], we dare to compute explicitly these triple contour integrals, but without any success, so we have written a computer program to do it for us. However there is a technical computational issue related to the evaluation of these triple contour integrals. Since the number of states involved in the \mathcal{L}_0 level expansion of the string field Ψ grows rapidly with the level, the complexity of performing many triple contour integrals, involved in the evaluation of the three point interaction vertex $\langle\Psi, \Psi * \Psi\rangle$, increases exponentially with the level. To solve this technical issue, it would be nice to find an alternative and efficient method to evaluate correlators involving the $*$ -product.

Apart from the technicality commented above, we also would like to mention that, in reference [27], we have derived expressions for correlation functions involving operators of the form $\hat{\mathcal{L}}^n\tilde{c}_p|0\rangle$ and $\hat{\mathcal{L}}^n\hat{\mathcal{B}}\tilde{c}_p\tilde{c}_q|0\rangle$. However we can have more general set of states, like the ones discussed in Schnabl's original paper [1], for instance states of the form

$$\hat{\mathcal{L}}^m\mathcal{L}_{n_1}\cdots\mathcal{L}_{n_k}\tilde{c}_p|0\rangle, \quad \text{with } n_1, \dots, n_k = -2, -3, \dots \quad (1.7)$$

$$\hat{\mathcal{L}}^m\hat{\mathcal{B}}\mathcal{L}_{n_1}\cdots\mathcal{L}_{n_k}\tilde{c}_p\tilde{c}_q|0\rangle, \quad \text{with } n_1, \dots, n_k = -2, -3, \dots \quad (1.8)$$

$$\hat{\mathcal{L}}^m\mathcal{L}_{n_1}\cdots\mathcal{L}_{n_k}\mathcal{B}_l\tilde{c}_p\tilde{c}_q|0\rangle, \quad \text{with } n_1, \dots, n_k, l = -2, -3, \dots \quad (1.9)$$

$$\hat{\mathcal{L}}^m\hat{\mathcal{B}}\mathcal{L}_{n_1}\cdots\mathcal{L}_{n_k}\mathcal{B}_l\tilde{c}_p\tilde{c}_q\tilde{c}_r|0\rangle, \quad \text{with } n_1, \dots, n_k, l = -2, -3, \dots \quad (1.10)$$

where the operators \mathcal{L}_{n_k} and \mathcal{B}_l are modes of the worldsheet energy momentum tensor and the b ghost in the \tilde{z} coordinate. By a conformal transformation they can be written as

$$\mathcal{L}_{n_k} = \oint \frac{dz}{2\pi i} (1+z^2)(\arctan z)^{n_k+1} T(z), \quad (1.11)$$

$$\mathcal{B}_l = \oint \frac{dz}{2\pi i} (1+z^2)(\arctan z)^{l+1} b(z). \quad (1.12)$$

At this point we would like to address the question: is there an alternative method for evaluating the three point interaction vertex which involves generic vertex operators like the ones discussed previously? We will show that the answer to this question turns out to be affirmative. This kind of problem also happens in the case where the string field is expanded in the usual Virasoro basis. In that case there are two ways of computing the interaction vertex: the first one is by means of finite conformal transformations of the three vertex operators, the second and more efficient method is by using conservation laws [50].

In order to implement this alternative method to compute the three point interaction vertex, in this paper we derive conservation laws in the case where the string field is expanded in the sliver \mathcal{L}_0 -basis. Schematically, in this method based on conservation laws, the task consists in finding (inside the three point correlation function) an expression for the operators $\hat{\mathcal{L}}^m$, $\hat{\mathcal{B}}$, \mathcal{L}_{n_k} and \mathcal{B}_l (with $n_k, l = -2, -3, \dots$) in terms of the modes \mathcal{L}_u and \mathcal{B}_v with $u, v = -1, 0, 1, \dots$. Since for these modes we have $\mathcal{L}_u|0\rangle = 0$, $\mathcal{B}_v|0\rangle = 0$, the commutation and anticommutation relations of \mathcal{L}_u and \mathcal{B}_v with the rest of operators allow to find a recursive procedure that express the three point correlation function of any state in terms of correlators which only involve modes of the c ghost. As an application of this new set of conservation laws, we compute the open string field action relevant to the tachyon condensation and in order to present not only an illustration but also an additional information, we evaluate the action without imposing a gauge choice.

This paper is organized as follows. In section 2, we review the straightforward techniques employed in the evaluation of correlation functions for operators defined in the sliver frame. In section 3, we discuss in detail the derivation of conservation laws for operators which are widely used in the \mathcal{L}_0 level expansion of the string field. In section 4, as an application of the conservation laws derived in the previous section, we compute the open string field action relevant to the tachyon condensation and in order to present not only an illustration but also an additional information, we evaluate the action without imposing a gauge choice. In section 5, a summary and further directions of exploration are given.

2 Correlation functions in the sliver frame

In this section, we will review the straightforward techniques used in the computation of correlation functions of operators defined in the sliver frame. Let us remark that the results shown in this section were derived in reference [27]. We begin with the description of the two and three string vertex which are used in the definition of the kinetic and cubic term of the open string field action.

2.1 The two and three string vertex

The relation between a point in the upper half plane (UHP) z and a point in the sliver frame \tilde{z} is given by the conformal map $\tilde{z} = \arctan z$. Note that by this conformal transformation, the UHP is mapped to a semi-infinite cylinder C_π of circumference π . It turns out that the sliver frame seems to be the most natural one since the conformal field theory in this new coordinate system remains easy. As in the case of the UHP, we can define general n -point correlation functions on C_π which can be readily found in terms of correlation functions defined on the UHP by a conformal mapping,

$$\langle \phi_1(\tilde{x}_1) \cdots \phi_n(\tilde{x}_n) \rangle_{C_\pi} = \langle \tilde{\phi}_1(\tilde{x}_1) \cdots \tilde{\phi}_n(\tilde{x}_n) \rangle_{UHP}, \quad (2.1)$$

where the fields $\tilde{\phi}_i(\tilde{x}_i)$ are defined as conformal transformation $\tilde{\phi}_i(\tilde{x}_i) = \tan \circ \phi_i(\tilde{x}_i)$. In general $f \circ \phi$ denotes a conformal transformation of a field ϕ under a map f , for instance if ϕ represents a primary field of dimension h , then $f \circ \phi$ is defined as $f \circ \phi(x) = (f'(x))^h \phi(f(x))$.

The two-string vertex which appears in the string field theory action is the familiar BPZ inner product of conformal field theory². It is defined as a map $\mathcal{H} \otimes \mathcal{H} \rightarrow \mathbb{R}$

$$\langle \phi_1, \phi_2 \rangle = \langle \mathcal{I} \circ \phi_1(0) \phi_2(0) \rangle_{UHP}, \quad (2.2)$$

where $\mathcal{I} : z \rightarrow -1/z$ is the inversion symmetry. For states defined on the sliver frame $|\tilde{\phi}_i\rangle$ the two-string vertex can be written as

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle = \langle \mathcal{I} \circ \tilde{\phi}_1(0) \tilde{\phi}_2(0) \rangle_{UHP} = \langle \phi_1(\frac{\pi}{2}) \phi_2(0) \rangle_{C_\pi}. \quad (2.3)$$

As we can see in this last expression, we evaluate the two-string vertex at two different points, namely at $\pi/2$ and 0 on C_π . This must be the case since the inversion symmetry maps the point at $z = 0$ on the upper half plane to the point at infinity, but the point at infinity is mapped to the point $\pm\pi/2$ on C_π .

The three-string vertex is a map $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \rightarrow \mathbb{R}$, and it is defined as a correlator on a surface formed by gluing together three strips representing three open strings. For states defined on the sliver frame $|\tilde{\phi}_i\rangle$ the three-string vertex can be written as

$$\langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle = \langle \phi_1(\frac{3\pi}{4}) \phi_2(\frac{\pi}{4}) \phi_3(-\frac{\pi}{4}) \rangle_{C_{\frac{3\pi}{2}}}. \quad (2.4)$$

Here the correlator is taken on a semi-infinite cylinder $C_{\frac{3\pi}{2}}$ of circumference $3\pi/2$. Also, this correlator can be evaluated on the semi-infinite cylinder C_π of circumference π . We only need to perform a simple conformal map (scaling) $s : \tilde{z} \rightarrow \frac{2}{3}\tilde{z}$ which brings the region $C_{\frac{3\pi}{2}}$ to C_π , and the correlator is given by

$$\langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle = \langle s \circ \phi_1(\frac{3\pi}{4}) s \circ \phi_2(\frac{\pi}{4}) s \circ \phi_3(-\frac{\pi}{4}) \rangle_{C_\pi}. \quad (2.5)$$

²Recall that the BPZ conjugate for the modes of an holomorphic field ϕ of dimension h is given by $\text{bpz}(\phi_n) = (-1)^{n+h} \phi_{-n}$.

Note that the scaling transformation s is implemented by $U_3 = (2/3)^{\mathcal{L}_0}$, where \mathcal{L}_0 is the zero mode of the worldsheet energy momentum tensor $T_{\tilde{z}\tilde{z}}(\tilde{z})$ in the \tilde{z} coordinate,

$$\mathcal{L}_0 = \oint \frac{d\tilde{z}}{2\pi i} \tilde{z} T_{\tilde{z}\tilde{z}}(\tilde{z}) . \quad (2.6)$$

By a conformal transformation it can be expressed as

$$\mathcal{L}_0 = \oint \frac{dz}{2\pi i} (1+z^2) \arctan z T_{zz}(z) = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k} , \quad (2.7)$$

where the L_n 's are the ordinary Virasoro generators (with zero central charge) of the full (matter plus ghost) conformal field theory.

2.2 Evaluating some correlators

Using the definition of the conformal transformation $\tilde{c}(x) = \cos^2(x)c(\tan x)$ of the c ghost and its anticommutation relations with the operators Q_B , \mathcal{B}_0 and B_1 ,³

$$\{Q_B, \tilde{c}(z)\} = \tilde{c}(z) \partial \tilde{c}(z) , \quad (2.8)$$

$$\{\mathcal{B}_0, \tilde{c}(z)\} = z , \quad (2.9)$$

$$\{B_1, \tilde{c}(z)\} = 1 , \quad (2.10)$$

we obtain the following basic correlation functions,

$$\langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle = \sin(x-y) \sin(x-z) \sin(y-z) , \quad (2.11)$$

$$\langle \tilde{c}(x) Q_B \tilde{c}(y) \rangle = -\sin(x-y)^2 , \quad (2.12)$$

$$\langle \tilde{c}(x) \mathcal{B}_0 \tilde{c}(y) \tilde{c}(z) \tilde{c}(w) \rangle = y \langle \tilde{c}(x) \tilde{c}(z) \tilde{c}(w) \rangle - z \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(w) \rangle + w \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle , \quad (2.13)$$

$$\langle \tilde{c}(x) \tilde{c}(y) \mathcal{B}_0 \tilde{c}(z) \tilde{c}(w) \rangle = z \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(w) \rangle - w \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle , \quad (2.14)$$

$$\langle \tilde{c}(x) B_1 \tilde{c}(y) \tilde{c}(z) \tilde{c}(w) \rangle = \langle \tilde{c}(x) \tilde{c}(z) \tilde{c}(w) \rangle - \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(w) \rangle + \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle , \quad (2.15)$$

$$\langle \tilde{c}(x) \tilde{c}(y) B_1 \tilde{c}(z) \tilde{c}(w) \rangle = \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(w) \rangle - \langle \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle . \quad (2.16)$$

In order to write, in a short way, long and messy expressions, the following definitions will be very useful

$$\begin{aligned} \sigma(a) &\equiv \oint \frac{dz}{2\pi i} z^a \sin(2z) \\ &= \frac{\theta(-a-2)}{\Gamma(-a)} ((-1)^a + 1) (-1)^{\frac{2-a}{2}} 2^{-a-2} , \end{aligned} \quad (2.17)$$

$$\begin{aligned} \varsigma(a) &\equiv \oint \frac{dz}{2\pi i} z^a \cos(2z) \\ &= \frac{\theta(-a-1)}{\Gamma(-a)} ((-1)^a - 1) (-1)^{\frac{1-a}{2}} 2^{-a-2} , \end{aligned} \quad (2.18)$$

³The operators \mathcal{B}_0 and $B_1 \equiv \mathcal{B}_{-1}$ are modes of the b ghost which are defined on the semi-infinite cylinder coordinate as $\mathcal{B}_n = \oint \frac{dz}{2\pi i} (1+z^2)(\arctan z)^{n+1} b(z)$.

$$\begin{aligned}
\mathcal{F}(a_1, a_2, a_3, \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3) &\equiv \oint \frac{dx_1 dx_2 dx_3}{(2\pi i)^3} x_1^{a_1} x_2^{a_2} x_3^{a_3} \langle \tilde{c}(\alpha_1 x_1 + \beta_1) \tilde{c}(\alpha_2 x_2 + \beta_2) \tilde{c}(\alpha_3 x_3 + \beta_3) \rangle \\
&= \frac{1}{\alpha_1^{a_1+1} \alpha_2^{a_2+1} \alpha_3^{a_3+1}} \left[\right. \\
&\delta_{a_3, -1} \frac{(\sigma(a_1)\sigma(a_2) + \varsigma(a_1)\varsigma(a_2)) \sin(2(\beta_1 - \beta_2)) + (\sigma(a_1)\varsigma(a_2) - \varsigma(a_1)\sigma(a_2)) \cos(2(\beta_1 - \beta_2))}{4} \\
&+ \delta_{a_2, -1} \frac{(\varsigma(a_1)\sigma(a_3) - \sigma(a_1)\varsigma(a_3)) \cos(2(\beta_1 - \beta_3)) - (\varsigma(a_1)\varsigma(a_3) + \sigma(a_1)\sigma(a_3)) \sin(2(\beta_1 - \beta_3))}{4} \\
&\left. + \delta_{a_1, -1} \frac{(\varsigma(a_2)\varsigma(a_3) + \sigma(a_2)\sigma(a_3)) \sin(2(\beta_2 - \beta_3)) + (\sigma(a_2)\varsigma(a_3) - \varsigma(a_2)\sigma(a_3)) \cos(2(\beta_2 - \beta_3))}{4} \right], \tag{2.19}
\end{aligned}$$

where $\theta(n)$ is the unit step (Heaviside) function which is defined as

$$\theta(n) = \begin{cases} 0, & \text{if } n < 0 \\ 1, & \text{if } n \geq 0. \end{cases} \tag{2.20}$$

Let us list some correlation functions involving operators frequently used in the \mathcal{L}_0 -basis, namely the operators $\hat{\mathcal{L}}^n$ ($\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger$), $\hat{\mathcal{B}}$ ($\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger$), $U_r = (\frac{2}{r})^{\mathcal{L}_0}$ and the $\tilde{c}(z)$ ghost

$$\begin{aligned}
\langle \text{bpz}(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \rangle &= \\
&= \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \langle \tilde{c}(x_1 + \frac{\pi}{2}) \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \rangle, \tag{2.21}
\end{aligned}$$

$$\begin{aligned}
\langle \text{bpz}(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1} \hat{\mathcal{B}} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \tilde{c}(z) \rangle &= \\
&= -\delta_{p_1, 0} \oint \frac{dz_1}{2\pi i} \frac{(-2)^{n_1} n_1!}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \langle \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \tilde{c}(\frac{4}{z_1 r} z) \rangle \\
&+ \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2} \langle \tilde{c}(x_1 + \frac{\pi}{2}) \mathcal{B}_0 \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \tilde{c}(\frac{4}{z_1 r} z) \rangle, \tag{2.22}
\end{aligned}$$

$$\begin{aligned}
\langle \text{bpz}(\tilde{c}_{p_1}) \text{bpz}(\tilde{c}_{p_2}) \hat{\mathcal{L}}^{n_1} \hat{\mathcal{B}} U_r^\dagger U_r \tilde{c}(x) \tilde{c}(y) \rangle &= \\
&= -\delta_{p_2, 0} \oint \frac{dz_1 dx_1}{(2\pi i)^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \langle \tilde{c}(x_1 + \frac{\pi}{2}) \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \rangle \\
&+ \delta_{p_1, 0} \oint \frac{dz_1 dx_2}{(2\pi i)^2} \frac{(-2)^{n_1} n_1! x_2^{p_2-2}}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \langle \tilde{c}(x_2 + \frac{\pi}{2}) \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \rangle \\
&+ \oint \frac{dz_1 dx_1 dx_2}{(2\pi i)^3} \frac{(-2)^{n_1} n_1! x_1^{p_1-2} x_2^{p_2-2}}{(z_1 - 2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1} \times \\
&\quad \times \langle \tilde{c}(x_1 + \frac{\pi}{2}) \tilde{c}(x_2 + \frac{\pi}{2}) \mathcal{B}_0 \tilde{c}(\frac{4}{z_1 r} x) \tilde{c}(\frac{4}{z_1 r} y) \rangle, \tag{2.23}
\end{aligned}$$

where the bpz acting on the modes of the $\tilde{c}(z)$ ghost stands for the usual BPZ conjugation which in the \mathcal{L}_0 -basis is defined as follows

$$bpz(\tilde{\phi}_n) = \oint \frac{dz}{2\pi i} z^{n+h-1} \tilde{\phi}(z + \frac{\pi}{2}), \quad (2.24)$$

for any primary field $\tilde{\phi}(z)$ with weight h . The action of the BPZ conjugation on the modes of $\tilde{\phi}(z)$ satisfies the following useful property

$$U_r^{\dagger -1} bpz(\tilde{\phi}_n) U_r^{\dagger} = \left(\frac{2}{r}\right)^{-n} bpz(\tilde{\phi}_n). \quad (2.25)$$

Correlation functions which involve only modes of the $\tilde{c}(z)$ ghost can be expressed in terms of the contour integral (2.19) as follows

$$\langle \tilde{c}_p \tilde{c}_q \tilde{c}_r \rangle = \mathcal{F}(p-2, q-2, r-2, 1, 0, 1, 0, 1, 0), \quad (2.26)$$

$$\langle bpz(\tilde{c}_p) \tilde{c}_q \tilde{c}_r \rangle = \mathcal{F}(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, 0, 1, 0), \quad (2.27)$$

$$\langle bpz(\tilde{c}_p) bpz(\tilde{c}_q) \tilde{c}_r \rangle = \mathcal{F}(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, \frac{\pi}{2}, 1, 0). \quad (2.28)$$

Correlators involving modes of the $\tilde{c}(z)$ ghost and insertions of operators $\hat{\mathcal{L}}^n$, $\hat{\mathcal{B}}$, can be obtained by using the basic correlators (2.13)-(2.16) and the definition of $\hat{\mathcal{L}}^n \equiv (-2)^n n! \oint \frac{dz}{2\pi i} \frac{1}{(z-2)^{n+1}} U_z^{\dagger} U_z$. For instance, let us write a correlator involving a $\hat{\mathcal{L}}^n$ insertion,

$$\langle bpz(\tilde{c}_p) (\mathcal{L}_0 + \mathcal{L}_0^{\dagger})^n \tilde{c}_q \tilde{c}_r \rangle = (-1)^n n! \binom{p+q+r}{n} \mathcal{F}(p-2, q-2, r-2, 1, \frac{\pi}{2}, 1, 0, 1, 0). \quad (2.29)$$

As it was shown in reference [27], the computations of correlators involving the star $*$ -product of states defined in the sliver frame are straightforward. For instance, let us compute the correlator $\langle 0 | bpz(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2} \tilde{c}_{p_2} | 0 \rangle * \hat{\mathcal{L}}^{n_3} \tilde{c}_{p_3} | 0 \rangle$,

$$\begin{aligned} & \langle 0 | bpz(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2} \tilde{c}_{p_2} | 0 \rangle * \hat{\mathcal{L}}^{n_3} \tilde{c}_{p_3} | 0 \rangle = \\ &= \frac{(-2)^{n_2+n_3} n_2! n_3!}{(2\pi i)^4} \oint \frac{dz_2 dz_3 dx_2 dx_3 x_2^{p_2-2} x_3^{p_3-2}}{(z_2-2)^{n_2+1} (z_3-2)^{n_3+1}} \langle 0 | bpz(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1}, U_{z_2}^{\dagger} U_{z_2} \tilde{c}(x_2) | 0 \rangle * U_{z_3}^{\dagger} U_{z_3} \tilde{c}(x_3) | 0 \rangle \\ &= \frac{(-2)^{n_2+n_3} n_2! n_3!}{(2\pi i)^4} \oint \frac{dz_2 dz_3 dx_2 dx_3 x_2^{p_2-2} x_3^{p_3-2}}{(z_2-2)^{n_2+1} (z_3-2)^{n_3+1}} \times \\ &\times \langle bpz(\tilde{c}_{p_1}) \hat{\mathcal{L}}^{n_1} U_r^{\dagger} U_r \tilde{c}(x_2 + \frac{\pi}{4}(z_3-1)) \tilde{c}(x_3 - \frac{\pi}{4}(z_2-1)) \rangle \\ &= \frac{(-1)^{n_1+n_2+n_3} 2^{2n_1+2n_2+2n_3-2p_1-4} n_1! n_2! n_3!}{(2\pi i)^3} \oint \frac{dz_1 dz_2 dz_3 z_1^{p_1+2} r^{p_1+2-n_1}}{(z_1-2)^{n_1+1} (z_2-2)^{n_2+1} (z_3-2)^{n_3+1}} \times \\ &\times \mathcal{F}(p_1-2, p_2-2, p_3-2, 1, \frac{\pi}{2}, \frac{4}{z_1 r}, \frac{\pi(z_3-1)}{z_1 r}, \frac{4}{z_1 r}, \frac{\pi(1-z_2)}{z_1 r}), \end{aligned} \quad (2.30)$$

where we have denoted $r \equiv z_2 + z_3 - 1$, and the function \mathcal{F} was defined in equation (2.19).

We have noted that all correlators involving the two point vertex, namely the BPZ inner product, can be expressed in terms of the function (2.19) evaluated at some particular points without performing any extra contour integral, e.g. equation (2.29). On the other hand correlators involving the three point interaction vertex, namely correlators involving the $*$ -product can be expressed in terms of triple contour integrals having as an integrand the function (2.19), e.g. equation (2.30). In reference [27], we dare to compute explicitly these triple contour integrals, but without any success, so we have written a computer program to do it for us. Nevertheless to evaluate the tachyon potential using a string field expanded in the \mathcal{L}_0 -basis there is a technical computational issue related to the evaluation of these triple contour integrals [27]. Since the number of states involved in the \mathcal{L}_0 level expansion of the string field Ψ grows exponentially with the level, the complexity of performing many triple contour integrals, involved in the evaluation of the three point interaction vertex $\langle \Psi, \Psi * \Psi \rangle$, increases exponentially as well. To solve this technical issue, it would be nice to find an alternative and efficient method to evaluate correlators involving the $*$ -product.

Let us also note that we have just derived a straightforward way of computing three point correlation functions involving operators of the form $\hat{\mathcal{L}}^n \tilde{c}_p |0\rangle$ and $\hat{\mathcal{L}}^n \hat{\mathcal{B}} \tilde{c}_p \tilde{c}_q |0\rangle$. However we can have more general set of states, like the ones discussed in Schnabl's original paper [1], for instance states of the form

$$\hat{\mathcal{L}}^m \mathcal{L}_{n_1} \cdots \mathcal{L}_{n_k} \tilde{c}_p |0\rangle, \quad \text{with } n_1, \dots, n_k = -2, -3, \dots \quad (2.31)$$

$$\hat{\mathcal{L}}^m \hat{\mathcal{B}} \mathcal{L}_{n_1} \cdots \mathcal{L}_{n_k} \tilde{c}_p \tilde{c}_q |0\rangle, \quad \text{with } n_1, \dots, n_k = -2, -3, \dots \quad (2.32)$$

$$\hat{\mathcal{L}}^m \mathcal{L}_{n_1} \cdots \mathcal{L}_{n_k} \mathcal{B}_l \tilde{c}_p \tilde{c}_q |0\rangle, \quad \text{with } n_1, \dots, n_k, l = -2, -3, \dots \quad (2.33)$$

$$\hat{\mathcal{L}}^m \hat{\mathcal{B}} \mathcal{L}_{n_1} \cdots \mathcal{L}_{n_k} \mathcal{B}_l \tilde{c}_p \tilde{c}_q \tilde{c}_r |0\rangle, \quad \text{with } n_1, \dots, n_k, l = -2, -3, \dots \quad (2.34)$$

where the operators \mathcal{L}_{n_k} and \mathcal{B}_l are modes of the worldsheet energy momentum tensor and the b ghost in the \tilde{z} coordinate (sliver frame). By a conformal transformation they can be written as

$$\mathcal{L}_{n_k} = \oint \frac{dz}{2\pi i} (1+z^2) (\arctan z)^{n_k+1} T(z), \quad (2.35)$$

$$\mathcal{B}_l = \oint \frac{dz}{2\pi i} (1+z^2) (\arctan z)^{l+1} b(z). \quad (2.36)$$

Based on conservation laws [50], we will derive an alternative method for computing the three point interaction vertex defined in the sliver frame. The advantage of this alternative method is that the vertex operators involved in the evaluation of the three point correlation function are generic ones, not just the operators discussed in reference [27] but also operators like the ones given in (2.31)-(2.34). Schematically, in this alternative method based on conservation laws, the task consists in finding (inside the three point correlation function) an expression for the operators $\hat{\mathcal{L}}^m$, $\hat{\mathcal{B}}$, \mathcal{L}_{n_k} and \mathcal{B}_l (with $n_k, l = -2, -3, \dots$) in terms of the modes \mathcal{L}_u and \mathcal{B}_v with $u, v = -1, 0, 1, \dots$. Since for these

modes we have $\mathcal{L}_u|0\rangle = 0$, $\mathcal{B}_v|0\rangle = 0$, the commutation and anticommutation relations of \mathcal{L}_u and \mathcal{B}_v with the rest of operators allow to find a recursive procedure that express the three point correlation function of any state in terms of correlators which only involve the modes of the c ghost.

3 Conservation laws

In this section, we will discuss in detail the derivation of conservation laws for operators which are widely used in the \mathcal{L}_0 level expansion of the string field. We begin by reviewing the derivation of the conservation laws of a rather generic conformal field and then apply the result to the case of operators defined on the sliver frame.

Suppose we have a conformal field ϕ of weight h . Let $v(z)$ be a field of conformal weight $1 - h$, then the quantity $v(z)\phi(z)dz$ transform as a 1-form. We require $v(z)$ to be holomorphic everywhere in the z plane, except at the punctures where it may have poles. For this 1-form $v(z)\phi(z)dz$ to be regular at infinity, $\lim_{z \rightarrow \infty} z^{2-2h}v(z)$ must be constant (or zero). Thanks to the holomorphicity of $v(z)$, integration contours in the z plane can be continuously deformed as long as we do not cross a puncture. Consider a contour \mathcal{C} which encircles the n punctures at $f_1^n(0), \dots, f_n^n(0)$. For arbitrary vertex operators Φ_i , the correlator

$$\left\langle \oint_{\mathcal{C}} \frac{1}{2\pi i} v(z)\phi(z)dz f_1^n \circ \Phi_1(0) \cdots f_n^n \circ \Phi_n(0) \right\rangle \quad (3.1)$$

vanishes identically, by shrinking the contour \mathcal{C} to zero size around the point at infinity.

Since the n -point correlation function (3.1) is zero for arbitrary vertex operators Φ_i , we can write

$$\langle V_n | \oint_{\mathcal{C}} \frac{1}{2\pi i} v(z)\phi(z)dz = 0. \quad (3.2)$$

Deforming the contour \mathcal{C} into the sum of n contours \mathcal{C}_i around the n punctures, and referring the 1-form to the local coordinates, we obtain the basic relation

$$\langle V_n | \sum_{i=1}^n \oint_{\mathcal{C}_i} \frac{1}{2\pi i} v^{(i)}(z_i)\phi(z_i)dz_i = 0, \quad (3.3)$$

where $v^{(i)}(z_i) = (\partial_{z_i} f_i^n(z_i))^{1-h} v(f_i^n(z_i))$. The notation $\langle V_n |$ represents the number of interaction vertices [50].

3.1 Virasoro conservation laws

Let us recall that the operators used in the \mathcal{L}_0 -basis are given in terms of the basic operators $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$ and \tilde{c}_p . These operators are related to the worldsheet energy momentum

tensor $T(z)$, the $b(z)$ and $c(z)$ ghosts fields respectively. In this subsection we will derive the conservation law for the $\hat{\mathcal{L}}$ operator

$$\hat{\mathcal{L}} = \oint \frac{dz}{2\pi i} (1+z^2)(\arctan z + \operatorname{arccot} z) T(z). \quad (3.4)$$

Since we are interested in the conservation laws of operators defined on the sliver frame. Using the conformal mapping $\tilde{z} = \arctan z$, we can write the expression of the $\hat{\mathcal{L}}$ operator as it is given in the sliver frame

$$\hat{\mathcal{L}} = \frac{\pi}{2} \oint \frac{d\tilde{z}}{2\pi i} \varepsilon(\operatorname{Re} \tilde{z}) \tilde{T}(\tilde{z}), \quad (3.5)$$

where $\varepsilon(x)$ is the step function equal to ± 1 for positive or negative values of its argument respectively.

For vertex operators Φ_i defined on the sliver frame, the three functions f_1 , f_2 and f_3 which appear in the definition of the three point vertex $\langle f_1 \circ \Phi_1(0) f_2 \circ \Phi_2(0) f_3 \circ \Phi_3(0) \rangle$ are given by

$$f_1(\tilde{z}_1) = \tan\left(\frac{\pi}{3} + \frac{2}{3}\tilde{z}_1\right), \quad (3.6)$$

$$f_2(\tilde{z}_2) = \tan\left(\frac{2}{3}\tilde{z}_2\right), \quad (3.7)$$

$$f_3(\tilde{z}_3) = \tan\left(-\frac{\pi}{3} + \frac{2}{3}\tilde{z}_3\right). \quad (3.8)$$

We are looking for conservation laws such that the operator $\hat{\mathcal{L}}$ acting on the three point interaction vertex $\langle V_3 |$ can be expressed in terms of positive Virasoro modes defined on the sliver frame⁴

$$\langle V_3 | \hat{\mathcal{L}}^{(2)} = \langle V_3 | \left(\sum_{n=0}^{\infty} a_n \mathcal{L}_n^{(1)} + \sum_{n=0}^{\infty} b_n \mathcal{L}_n^{(2)} + \sum_{n=0}^{\infty} c_n \mathcal{L}_n^{(3)} \right), \quad (3.9)$$

where a_n , b_n and c_n are coefficients that will be determined below and depend on the geometry of the vertex. By the cyclicity property of the three vertex, the same identity (3.9) holds after letting $(1) \rightarrow (2)$, $(2) \rightarrow (3)$, $(3) \rightarrow (1)$.

In order to find conservation laws of the form (3.9), we need a vector field which behaves as $v^{(2)}(\tilde{z}_2) \sim \frac{\pi}{2}\varepsilon(\operatorname{Re} \tilde{z}_2) + O(\tilde{z}_2)$ around puncture 2, and has the following behavior in the other two punctures, $v^{(1)}(\tilde{z}_1) \sim O(\tilde{z}_3)$, $v^{(3)}(\tilde{z}_3) \sim O(\tilde{z}_3)$. A vector field which does this job is given by

$$v(z) = \frac{2}{3}(1+z^2)\operatorname{arccot} z - \frac{4\pi}{9\sqrt{3}}z. \quad (3.10)$$

⁴We are going to use the following notation $\mathcal{O}^{(i)}$ to refer an operator \mathcal{O} defined around the i -th puncture.

This vector field is not globally defined, but is holomorphic everywhere outside the unit circle including the infinity, so that (3.1) still holds for a contour encircling the infinity.

Using equations (3.6)-(3.8) and (3.10) into the definition $v^{(i)}(\tilde{z}_i) = (\partial_{\tilde{z}_i} f_i(\tilde{z}_i))^{-1} v(f_i(\tilde{z}_i))$ of the vector fields $v^{(1)}(\tilde{z}_1)$, $v^{(2)}(\tilde{z}_2)$ and $v^{(3)}(\tilde{z}_3)$, we find that

$$v^{(1)}(\tilde{z}_1) = \left(-\frac{2}{3} + \frac{2\pi}{9\sqrt{3}}\right)\tilde{z}_1 + \frac{4\pi\tilde{z}_1^2}{27} - \frac{16\pi\tilde{z}_1^3}{243\sqrt{3}} - \frac{16\pi\tilde{z}_1^4}{729} + \frac{64\pi\tilde{z}_1^5}{10935\sqrt{3}} + O(\tilde{z}_1^6), \quad (3.11)$$

$$v^{(2)}(\tilde{z}_2) = \frac{\pi}{2}\varepsilon(\text{Re}\tilde{z}_2) + \left(-\frac{2}{3} - \frac{4\pi}{9\sqrt{3}}\right)\tilde{z}_2 + \frac{32\pi\tilde{z}_2^3}{243\sqrt{3}} - \frac{128\pi\tilde{z}_2^5}{10935\sqrt{3}} + O(\tilde{z}_2^7), \quad (3.12)$$

$$v^{(3)}(\tilde{z}_3) = \left(-\frac{2}{3} + \frac{2\pi}{9\sqrt{3}}\right)\tilde{z}_3 - \frac{4\pi\tilde{z}_3^2}{27} - \frac{16\pi\tilde{z}_3^3}{243\sqrt{3}} + \frac{16\pi\tilde{z}_3^4}{729} + \frac{64\pi\tilde{z}_3^5}{10935\sqrt{3}} + O(\tilde{z}_3^6). \quad (3.13)$$

Due to the presence of the step function we see that the vector field $v^{(2)}(\tilde{z}_2)$ is discontinuous around puncture 2, since we are interesting in the conservation law of the operator defined in equation (3.5), this kind of discontinuity is what we want. Using (3.3) and noting that integration amounts to the replacement $v_n^{(i)} \tilde{z}_i^n \rightarrow v_n^{(i)} \mathcal{L}_{n-1}^{(i)}$, we can immediately write the conservation law

$$\begin{aligned} 0 = & \langle V_3 | \left(\left(-\frac{2}{3} + \frac{2\pi}{9\sqrt{3}} \right) \mathcal{L}_0 + \frac{4\pi}{27} \mathcal{L}_1 - \frac{16\pi}{243\sqrt{3}} \mathcal{L}_2 - \frac{16\pi}{729} \mathcal{L}_3 + \frac{64\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\hat{\mathcal{L}} + \left(-\frac{2}{3} - \frac{4\pi}{9\sqrt{3}} \right) \mathcal{L}_0 + \frac{32\pi}{243\sqrt{3}} \mathcal{L}_2 - \frac{128\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(\left(-\frac{2}{3} + \frac{2\pi}{9\sqrt{3}} \right) \mathcal{L}_0 - \frac{4\pi}{27} \mathcal{L}_1 - \frac{16\pi}{243\sqrt{3}} \mathcal{L}_2 + \frac{16\pi}{729} \mathcal{L}_3 + \frac{64\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(3)}. \end{aligned} \quad (3.14)$$

Thanks to the cyclicity of the string vertex, analogous identities hold by cycling the punctures, (1) \rightarrow (2), (2) \rightarrow (3), (3) \rightarrow (1).

Let us recall that the \mathcal{L}_0 -basis contains operators of the form $\hat{\mathcal{L}}^N$ with $N = 1, 2, \dots$. At this point we just have shown the conservation law for the $\hat{\mathcal{L}}$ operator (3.14), namely the $N = 1$ case. It remains to find conservation laws for the other values of N , for instance the $N = 2$ case reads

$$\begin{aligned} \langle V_3 | \hat{\mathcal{L}}^{(2)} \hat{\mathcal{L}}^{(2)} &= \langle V_3 | \left(\sum_{n=0}^{\infty} a_n \mathcal{L}_n^{(1)} + \sum_{n=0}^{\infty} b_n \mathcal{L}_n^{(2)} + \sum_{n=0}^{\infty} c_n \mathcal{L}_n^{(3)} \right) \hat{\mathcal{L}}^{(2)} \\ &= \langle V_3 | \left(\sum_{n=0}^{\infty} a_n \mathcal{L}_n^{(1)} + \sum_{n=0}^{\infty} b_n \mathcal{L}_n^{(2)} + \sum_{n=0}^{\infty} c_n \mathcal{L}_n^{(3)} \right)^2 + \\ &\quad \langle V_3 | \left[\left(\sum_{n=0}^{\infty} a_n \mathcal{L}_n^{(1)} + \sum_{n=0}^{\infty} b_n \mathcal{L}_n^{(2)} + \sum_{n=0}^{\infty} c_n \mathcal{L}_n^{(3)} \right), \hat{\mathcal{L}}^{(2)} \right]. \end{aligned} \quad (3.15)$$

From equation (3.15) it is clear that for each $n \geq 0$, we will have to compute additional conservation laws of the form

$$\langle V_3 | [\mathcal{L}_n^{(i)}, \hat{\mathcal{L}}^{(2)}] = \langle V_3 | \left(\sum_{k=0}^{\infty} \alpha_{n,k}^{(i)} \mathcal{L}_k^{(1)} + \sum_{k=0}^{\infty} \beta_{n,k}^{(i)} \mathcal{L}_k^{(2)} + \sum_{k=0}^{\infty} \gamma_{n,k}^{(i)} \mathcal{L}_k^{(3)} \right). \quad (3.16)$$

Plugging this equation (3.16) into the last term of equation (3.15) produces double sums (running over the indices n and k). Following the same procedure as for the $N = 2$ case, for the $N = 3$ case we will get triple sums, for the $N = 4$ case four-fold sums and so on and so forth. Therefore due to the presence of these N -fold sums, it seems that this alternative method based on conservation laws should not be efficient for computing three point correlation functions involving operators of the form $\hat{\mathcal{L}}^N$. Nevertheless, as a remarkable result, in what follows, we will show that these N -fold sums can be avoided.

The idea is to find a conservation law which contains almost all the terms given in equation (3.14), except for some finite terms. Using equations (3.6)-(3.8) and the vector field $\hat{v}(z) = 4\pi z/(9\sqrt{3})$ into the definition $\hat{v}^{(i)}(\tilde{z}_i) = (\partial_{\tilde{z}_i} f_i(\tilde{z}_i))^{-1} \hat{v}(f_i(\tilde{z}_i))$ of the vector fields $\hat{v}^{(1)}(\tilde{z}_1)$, $\hat{v}^{(2)}(\tilde{z}_2)$ and $\hat{v}^{(3)}(\tilde{z}_3)$, we find that

$$\hat{v}^{(1)}(\tilde{z}_1) = \frac{\pi}{6} - \frac{2\pi\tilde{z}_1}{9\sqrt{3}} - \frac{4\pi\tilde{z}_1^2}{27} + \frac{16\pi\tilde{z}_1^3}{243\sqrt{3}} + \frac{16\pi\tilde{z}_1^4}{729} - \frac{64\pi\tilde{z}_1^5}{10935\sqrt{3}} + O(\tilde{z}_1^6), \quad (3.17)$$

$$\hat{v}^{(2)}(\tilde{z}_2) = \frac{4\pi\tilde{z}_2}{9\sqrt{3}} - \frac{32\pi\tilde{z}_2^2}{243\sqrt{3}} + \frac{128\pi\tilde{z}_2^3}{10935\sqrt{3}} + O(\tilde{z}_2^4), \quad (3.18)$$

$$\hat{v}^{(3)}(\tilde{z}_3) = -\frac{\pi}{6} - \frac{2\pi\tilde{z}_3}{9\sqrt{3}} + \frac{4\pi\tilde{z}_3^2}{27} + \frac{16\pi\tilde{z}_3^3}{243\sqrt{3}} - \frac{16\pi\tilde{z}_3^4}{729} - \frac{64\pi\tilde{z}_3^5}{10935\sqrt{3}} + O(\tilde{z}_3^6). \quad (3.19)$$

Using (3.3) and noting that integration amounts to the replacement $\hat{v}_n^{(i)} \tilde{z}_i^n \rightarrow \hat{v}_n^{(i)} \mathcal{L}_{n-1}^{(i)}$, we can immediately write the conservation law

$$\begin{aligned} 0 = & \langle V_3 | \left(\frac{\pi}{6} \mathcal{L}_{-1} - \frac{2\pi}{9\sqrt{3}} \mathcal{L}_0 - \frac{4\pi}{27} \mathcal{L}_1 + \frac{16\pi}{243\sqrt{3}} \mathcal{L}_2 + \frac{16\pi}{729} \mathcal{L}_3 - \frac{64\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\frac{4\pi}{9\sqrt{3}} \mathcal{L}_0 - \frac{32\pi}{243\sqrt{3}} \mathcal{L}_2 + \frac{128\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(-\frac{\pi}{6} \mathcal{L}_{-1} - \frac{2\pi}{9\sqrt{3}} \mathcal{L}_0 + \frac{4\pi}{27} \mathcal{L}_1 + \frac{16\pi}{243\sqrt{3}} \mathcal{L}_2 - \frac{16\pi}{729} \mathcal{L}_3 - \frac{64\pi}{10935\sqrt{3}} \mathcal{L}_4 + \dots \right)^{(3)}. \end{aligned} \quad (3.20)$$

Adding the equations (3.20) and (3.14), we obtain the following conservation law for the $\hat{\mathcal{L}}$ operator

$$\langle V_3 | \hat{\mathcal{L}}^{(2)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(3)}) + \frac{\pi}{6} \mathcal{L}_{-1}^{(3)} - \frac{\pi}{6} \mathcal{L}_{-1}^{(1)} \right). \quad (3.21)$$

Thanks to the cyclicity of the string vertex, analogous identities hold by cycling the punctures, $(1) \rightarrow (2)$, $(2) \rightarrow (3)$, $(3) \rightarrow (1)$

$$\langle V_3 | \hat{\mathcal{L}}^{(1)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(3)}) + \frac{\pi}{6} \mathcal{L}_{-1}^{(2)} - \frac{\pi}{6} \mathcal{L}_{-1}^{(3)} \right), \quad (3.22)$$

$$\langle V_3 | \hat{\mathcal{L}}^{(3)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(3)}) + \frac{\pi}{6} \mathcal{L}_{-1}^{(1)} - \frac{\pi}{6} \mathcal{L}_{-1}^{(2)} \right). \quad (3.23)$$

Since the conservation law (3.21) involves a finite numbers of terms containing the $\mathcal{L}_0^{(i)}$ and $\mathcal{L}_{-1}^{(i)}$ operators, using the commutators $[\mathcal{L}_0^{(i)}, \hat{\mathcal{L}}^{(j)}] = \delta^{ij} \hat{\mathcal{L}}^{(j)}$ and $[\mathcal{L}_{-1}^{(i)}, \hat{\mathcal{L}}^{(j)}] = 0$,

we can easily derive conservation laws for the operators $\hat{\mathcal{L}}^N$ with $N \geq 2$. For instance the $N = 2$ case reads

$$\begin{aligned} \langle V_3 | \hat{\mathcal{L}}^{(2)} \hat{\mathcal{L}}^{(2)} &= \langle V_3 | \left(\frac{2}{3} (\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(3)}) + \frac{\pi}{6} \mathcal{L}_{-1}^{(3)} - \frac{\pi}{6} \mathcal{L}_{-1}^{(1)} \right)^2 + \\ &\quad \langle V_3 | \left(\frac{4}{9} (\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(3)}) + \frac{\pi}{9} \mathcal{L}_{-1}^{(3)} - \frac{\pi}{9} \mathcal{L}_{-1}^{(1)} \right). \end{aligned} \quad (3.24)$$

Again due to the cyclicity of the string vertex, analogous identities hold by cycling the punctures, $(1) \rightarrow (2)$, $(2) \rightarrow (3)$, $(3) \rightarrow (1)$. Given the simplicity of the conservation law (3.24) compared to the one given in equation (3.15), we observe that, based on conservation laws, we can get an efficient and alternative method for computing three point correlation functions involving operators of the form $\hat{\mathcal{L}}^N$.

For completeness reasons, we also write conservation laws for some negative modes \mathcal{L}_{-k} with $k = 2, 3, \dots$ of the Virasoro operators defined on the sliver frame. With

$$v_2(z) = -\frac{4(z^2 - 3)}{27z}, \quad (3.25)$$

$$v_3(z) = -\frac{8(z^2 - 3)}{81z^2} - \frac{16}{81}(z^2 - 3), \quad (3.26)$$

$$v_4(z) = -\frac{16(z^2 - 3)}{243z^3} + \frac{28}{27}v_2(z), \quad (3.27)$$

we obtain the conservation laws

$$\begin{aligned} 0 = & \langle V_3 | \left(-\frac{8}{27}\mathcal{L}_0 + \frac{80}{81\sqrt{3}}\mathcal{L}_1 - \frac{64}{243}\mathcal{L}_2 + \frac{64}{729\sqrt{3}}\mathcal{L}_3 - \frac{5888}{98415}\mathcal{L}_4 + \frac{512}{6561\sqrt{3}}\mathcal{L}_5 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\mathcal{L}_{-2} - \frac{20}{27}\mathcal{L}_0 + \frac{208}{1215}\mathcal{L}_2 - \frac{2176}{137781}\mathcal{L}_4 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(-\frac{8}{27}\mathcal{L}_0 - \frac{80}{81\sqrt{3}}\mathcal{L}_1 - \frac{64}{243}\mathcal{L}_2 - \frac{64}{729\sqrt{3}}\mathcal{L}_3 - \frac{5888}{98415}\mathcal{L}_4 - \frac{512}{6561\sqrt{3}}\mathcal{L}_5 + \dots \right)^{(3)}. \end{aligned} \quad (3.28)$$

$$\begin{aligned} 0 = & \langle V_3 | \left(-\frac{112}{81\sqrt{3}}\mathcal{L}_0 + \frac{160}{243}\mathcal{L}_1 - \frac{128}{243\sqrt{3}}\mathcal{L}_2 + \frac{6272}{19683}\mathcal{L}_3 - \frac{14848}{32805\sqrt{3}}\mathcal{L}_4 + \frac{11264}{59049}\mathcal{L}_5 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\mathcal{L}_{-3} - \frac{304}{1215}\mathcal{L}_1 + \frac{5504}{137781}\mathcal{L}_3 - \frac{3328}{1476225}\mathcal{L}_5 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(\frac{112}{81\sqrt{3}}\mathcal{L}_0 + \frac{160}{243}\mathcal{L}_1 + \frac{128}{243\sqrt{3}}\mathcal{L}_2 + \frac{6272}{19683}\mathcal{L}_3 + \frac{14848}{32805\sqrt{3}}\mathcal{L}_4 + \frac{11264}{59049}\mathcal{L}_5 + \dots \right)^{(3)}. \end{aligned} \quad (3.29)$$

$$\begin{aligned} 0 = & \langle V_3 | \left(-\frac{256}{729}\mathcal{L}_0 + \frac{1024}{729\sqrt{3}}\mathcal{L}_1 - \frac{14336}{19683}\mathcal{L}_2 + \frac{20480}{19683\sqrt{3}}\mathcal{L}_3 - \frac{499712}{885735}\mathcal{L}_4 + \frac{6651904}{7971615\sqrt{3}}\mathcal{L}_5 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\mathcal{L}_{-4} - \frac{1328}{3645}\mathcal{L}_0 + \frac{14080}{137781}\mathcal{L}_2 - \frac{41728}{4428675}\mathcal{L}_4 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(-\frac{256}{729}\mathcal{L}_0 - \frac{1024}{729\sqrt{3}}\mathcal{L}_1 - \frac{14336}{19683}\mathcal{L}_2 - \frac{20480}{19683\sqrt{3}}\mathcal{L}_3 - \frac{499712}{885735}\mathcal{L}_4 - \frac{6651904}{7971615\sqrt{3}}\mathcal{L}_5 + \dots \right)^{(3)}. \end{aligned} \quad (3.30)$$

Thanks to the cyclicity of the string vertex, analogous identities hold by cycling the punctures, $(1) \rightarrow (2)$, $(2) \rightarrow (3)$, $(3) \rightarrow (1)$.

3.2 Ghost conservation laws

Since the b ghost is a conformal field of dimension two, the conservation laws for operators involving this field are identical to those for the Virasoro operators. For instance, the conservation laws for the operator $\hat{\mathcal{B}}$ reads

$$\langle V_3 | \hat{\mathcal{B}}^{(1)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{B}_0^{(1)} + \mathcal{B}_0^{(2)} + \mathcal{B}_0^{(3)}) + \frac{\pi}{6} \mathcal{B}_{-1}^{(2)} - \frac{\pi}{6} \mathcal{B}_{-1}^{(3)} \right), \quad (3.31)$$

$$\langle V_3 | \hat{\mathcal{B}}^{(2)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{B}_0^{(1)} + \mathcal{B}_0^{(2)} + \mathcal{B}_0^{(3)}) + \frac{\pi}{6} \mathcal{B}_{-1}^{(3)} - \frac{\pi}{6} \mathcal{B}_{-1}^{(1)} \right), \quad (3.32)$$

$$\langle V_3 | \hat{\mathcal{B}}^{(3)} = \langle V_3 | \left(\frac{2}{3} (\mathcal{B}_0^{(1)} + \mathcal{B}_0^{(2)} + \mathcal{B}_0^{(3)}) + \frac{\pi}{6} \mathcal{B}_{-1}^{(1)} - \frac{\pi}{6} \mathcal{B}_{-1}^{(2)} \right). \quad (3.33)$$

Therefore it remains the derivation of conservation laws for the c ghost. The c ghost is a primary field of dimension minus one, thus to derive its conservation laws we need to consider a globally defined quadratic differential

$$\varphi(z)(dz)^2 = \varphi(z')(dz')^2, \quad (3.34)$$

holomorphic everywhere except for possible poles at the punctures. Regularity at infinity requires the $\lim_{z \rightarrow \infty} z^4 \varphi(z)$ to be finite. The product $c(z)\varphi(z)dz$ is a 1-form, we can then use contour deformations and following exactly the same procedure as for the 1-form considered in the previous subsection, we derive

$$\langle V_3 | \sum_{i=1}^3 \oint_{\mathcal{C}_i} \frac{1}{2\pi i} \varphi^{(i)}(\tilde{z}_i) \tilde{c}(\tilde{z}_i) d\tilde{z}_i = 0, \quad (3.35)$$

where $\varphi^{(i)}(\tilde{z}_i) = (\partial_{\tilde{z}_i} f_i(\tilde{z}_i))^2 \varphi(f_i(\tilde{z}_i))$, and since we are considering conservation laws for operators defined on the sliver frame, the functions f_1 , f_2 and f_3 are the same as the ones given in equations (3.6)-(3.8).

For instance, with the quadratic differential

$$\varphi_0(z) = -\frac{3}{z^2(z^2 - 3)}, \quad (3.36)$$

using the definition $\varphi_0^{(i)}(\tilde{z}_i) = (\partial_{\tilde{z}_i} f_i(\tilde{z}_i))^2 \varphi_0(f_i(\tilde{z}_i))$ from equation (3.35), we derive the conservation law

$$\begin{aligned} 0 = & \langle V_3 | \left(-\frac{4}{3\sqrt{3}} \tilde{c}_1 + \frac{8}{27} \tilde{c}_2 - \frac{112}{81\sqrt{3}} \tilde{c}_3 + \frac{256}{729} \tilde{c}_4 - \frac{3392}{3645\sqrt{3}} \tilde{c}_5 + \frac{5120}{19683} \tilde{c}_6 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\tilde{c}_0 + \frac{20}{27} \tilde{c}_2 + \frac{1328}{3645} \tilde{c}_4 + \frac{21632}{137781} \tilde{c}_6 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(\frac{4}{3\sqrt{3}} \tilde{c}_1 + \frac{8}{27} \tilde{c}_2 + \frac{112}{81\sqrt{3}} \tilde{c}_3 + \frac{256}{729} \tilde{c}_4 + \frac{3392}{3645\sqrt{3}} \tilde{c}_5 + \frac{5120}{19683} \tilde{c}_6 + \dots \right)^{(3)}. \end{aligned} \quad (3.37)$$

Conservation laws for higher negative modes of the c ghost are obtained with quadratic differentials having higher order poles at $z = 0$. With

$$\varphi_1(z) = -\frac{2}{z^3(z^2-3)}, \quad (3.38)$$

$$\varphi_2(z) = -\frac{4}{3z^4(z^2-3)} - \frac{4}{9}\varphi_0(z), \quad (3.39)$$

$$\varphi_3(z) = -\frac{8}{9z^5(z^2-3)} - \frac{8}{27}\varphi_1(z), \quad (3.40)$$

we obtain the conservation laws

$$\begin{aligned} 0 = & \langle V_3 | \left(-\frac{8}{27}\tilde{c}_1 + \frac{80}{81\sqrt{3}}\tilde{c}_2 - \frac{160}{243}\tilde{c}_3 + \frac{1024}{729\sqrt{3}}\tilde{c}_4 - \frac{71552}{98415}\tilde{c}_5 + \frac{8192}{6561\sqrt{3}}\tilde{c}_6 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\tilde{c}_{-1} + \frac{16}{27}\tilde{c}_1 + \frac{304}{1215}\tilde{c}_3 + \frac{68608}{688905}\tilde{c}_5 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(-\frac{8}{27}\tilde{c}_1 - \frac{80}{81\sqrt{3}}\tilde{c}_2 - \frac{160}{243}\tilde{c}_3 - \frac{1024}{729\sqrt{3}}\tilde{c}_4 - \frac{71552}{98415}\tilde{c}_5 - \frac{8192}{6561\sqrt{3}}\tilde{c}_6 + \dots \right)^{(3)}. \end{aligned} \quad (3.41)$$

$$\begin{aligned} 0 = & \langle V_3 | \left(\frac{32}{81\sqrt{3}}\tilde{c}_1 + \frac{64}{243}\tilde{c}_2 - \frac{128}{243\sqrt{3}}\tilde{c}_3 + \frac{14336}{19683}\tilde{c}_4 - \frac{45568}{32805\sqrt{3}}\tilde{c}_5 + \frac{57344}{59049}\tilde{c}_6 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\tilde{c}_{-2} - \frac{208}{1215}\tilde{c}_2 - \frac{14080}{137781}\tilde{c}_4 - \frac{13568}{295245}\tilde{c}_6 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(-\frac{32}{81\sqrt{3}}\tilde{c}_1 + \frac{64}{243}\tilde{c}_2 + \frac{128}{243\sqrt{3}}\tilde{c}_3 + \frac{14336}{19683}\tilde{c}_4 + \frac{45568}{32805\sqrt{3}}\tilde{c}_5 + \frac{57344}{59049}\tilde{c}_6 + \dots \right)^{(3)}. \end{aligned} \quad (3.42)$$

$$\begin{aligned} 0 = & \langle V_3 | \left(\frac{32}{729}\tilde{c}_1 + \frac{64}{729\sqrt{3}}\tilde{c}_2 - \frac{6272}{19683}\tilde{c}_3 + \frac{20480}{19683\sqrt{3}}\tilde{c}_4 - \frac{833024}{885735}\tilde{c}_5 + \frac{3276800}{1594323\sqrt{3}}\tilde{c}_6 + \dots \right)^{(1)} + \\ & + \langle V_3 | \left(\tilde{c}_{-3} - \frac{64}{729}\tilde{c}_1 - \frac{5504}{137781}\tilde{c}_3 - \frac{13568}{885735}\tilde{c}_5 + \dots \right)^{(2)} + \\ & + \langle V_3 | \left(\frac{32}{729}\tilde{c}_1 - \frac{64}{729\sqrt{3}}\tilde{c}_2 - \frac{6272}{19683}\tilde{c}_3 - \frac{20480}{19683\sqrt{3}}\tilde{c}_4 - \frac{833024}{885735}\tilde{c}_5 - \frac{3276800}{1594323\sqrt{3}}\tilde{c}_6 + \dots \right)^{(3)}. \end{aligned} \quad (3.43)$$

Due to the cyclicity of the string vertex, analogous identities hold by cycling the punctures, $(1) \rightarrow (2)$, $(2) \rightarrow (3)$, $(3) \rightarrow (1)$. In the next section, we will give some illustrations of the use of these conservation laws for the calculation of the three point interaction vertex in open bosonic string field theory.

4 Tachyon potentials

As an application of the conservation laws derived in the previous section, we will compute here the open string field action relevant to the tachyon condensation and in order to

present not only an illustration but also an additional information, we evaluate the action without imposing a gauge choice.

4.1 Three point correlation function

Let us begin by introducing some notations. Given three operators defined in the sliver frame, we write the corresponding states by $\tilde{\phi}_1|0\rangle$, $\tilde{\phi}_2|0\rangle$ and $\tilde{\phi}_3|0\rangle$. The three point correlation function involving these operators will be denoted by

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 * \tilde{\phi}_3 \rangle = \langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle = \langle V_3 | \tilde{\phi}_1^{(1)} \tilde{\phi}_2^{(2)} \tilde{\phi}_3^{(3)} | 0 \rangle_{(3)} \otimes | 0 \rangle_{(2)} \otimes | 0 \rangle_{(1)}. \quad (4.1)$$

The basic three point correlation function that we will need is

$$\langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = \langle V_3 | \tilde{c}_1^{(1)} \tilde{c}_1^{(2)} \tilde{c}_1^{(3)} | 0 \rangle_{(3)} \otimes | 0 \rangle_{(2)} \otimes | 0 \rangle_{(1)}. \quad (4.2)$$

To compute this correlator, we use equation (2.4)

$$\begin{aligned} \langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle &= \langle c(\frac{3\pi}{4}) c(\frac{\pi}{4}) c(-\frac{\pi}{4}) \rangle_{C_{\frac{3\pi}{2}}} \\ &= \frac{3^3}{2^3} \sin^2\left(\frac{\pi}{3}\right) \sin\left(\frac{2\pi}{3}\right) = \frac{3^4 \sqrt{3}}{2^6}, \end{aligned} \quad (4.3)$$

where we have used the expression for the correlator $\langle c(x_1) c(x_2) c(x_3) \rangle_{C_L}$ given in [30]

$$\langle c(x_1) c(x_2) c(x_3) \rangle_{C_L} = \frac{L^3}{\pi^3} \sin\left(\frac{\pi(x_1 - x_2)}{L}\right) \sin\left(\frac{\pi(x_1 - x_3)}{L}\right) \sin\left(\frac{\pi(x_2 - x_3)}{L}\right). \quad (4.4)$$

The conservation laws derived in the previous section allow the computation of all necessary three point functions in terms of the basic correlator (4.3).

Since we will analyze the tachyon potential by performing computations in the \mathcal{L}_0 level truncation, we are going to define the level of a state as the eigenvalue of the operator $N = \mathcal{L}_0 + 1$. This definition is adjusted so that the zero momentum tachyon defined by the state $\tilde{c}_1|0\rangle$ is at level zero.

Having defined the level number of states contained in the level expansion of the string field, level of each term in the action is also defined to be the sum of the levels of the fields involved. For instance, if states $\tilde{\phi}_1$, $\tilde{\phi}_2$, $\tilde{\phi}_3$ have level N_1 , N_2 , N_3 respectively, we assign level $N_1 + N_2 + N_3$ to the interaction term $\langle \tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3 \rangle$. When we say level (M, N) , we mean that the string field includes all terms with level $\leq M$ while the action includes all terms with level $\leq N$.

The string field Ψ expanded in the sliver \mathcal{L}_0 -basis reads as follows

$$\Psi = \sum_{i=0}^{\infty} x_i |\psi^i\rangle, \quad (4.5)$$

where the state $|\psi^i\rangle$ is built by applying the modes of the $\tilde{c}(z)$ ghost and the operators $(\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \equiv \hat{\mathcal{L}}^n$, $\mathcal{B}_0 + \mathcal{B}_0^\dagger \equiv \hat{\mathcal{B}}$ on the $SL(2, \mathbb{R})$ invariant vacuum $|0\rangle$. The first term in the expansion is given by the zero-momentum tachyon $|\psi^0\rangle = \tilde{c}_1|0\rangle$. We will restrict our attention to an even-twist and ghost-number one string field Ψ . The tachyon potential we want to evaluate is defined as

$$V = 2\pi^2 \left[\frac{1}{2} \langle \Psi, Q_B \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle \right]. \quad (4.6)$$

4.2 The tachyon potential in arbitrary gauge

In order to illustrate the use of conservation laws, let us compute in detail the level (2,6) tachyon potential in arbitrary gauge. Expanding the string field up to level two states, we have

$$\Psi = x_0 \tilde{c}_1|0\rangle + x_1 \hat{\mathcal{L}} \tilde{c}_1|0\rangle + x_2 \hat{\mathcal{B}} \tilde{c}_0 \tilde{c}_1|0\rangle + x_3 \tilde{c}_{-1}|0\rangle + x_4 \hat{\mathcal{L}}^2 \tilde{c}_1|0\rangle + x_5 \hat{\mathcal{B}} \hat{\mathcal{L}} \tilde{c}_0 \tilde{c}_1|0\rangle. \quad (4.7)$$

The evaluation of the kinetic term $V_{\text{kin}} = \frac{1}{2} \langle \Psi, Q_B \Psi \rangle$ requires the action of the BRST operator and the computation of the BPZ inner product. For the string field given in equation (4.7), to the required level the computation of the kinetic term is straightforward, we find

$$V_{\text{kin}} = -\frac{x_0^2}{2} + 2x_1x_0 - 2x_2x_0 + x_3x_0 - 2x_4x_0 + 2x_5x_0 - x_1^2 - x_2^2 - x_3^2 + 2x_1x_2. \quad (4.8)$$

The real task is the evaluation of the cubic interaction term $V_{\text{inter}} = \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle$. Using the conservation laws derived in the previous section, we will illustrate the computation of the various cubic couplings. For instance, plugging (4.7) into the definition of the cubic interaction term, we get a coupling like

$$\langle \tilde{c}_1, \tilde{c}_{-1}, \tilde{c}_1 \rangle = \langle V_3 | \tilde{c}_1^{(1)} \tilde{c}_{-1}^{(2)} \tilde{c}_1^{(3)} | 0 \rangle_{(3)} \otimes | 0 \rangle_{(2)} \otimes | 0 \rangle_{(1)}, \quad (4.9)$$

where the subscripts denote the labels distinguishing the three state spaces. We want to relate this term (4.9) to the correlator given in equation (4.3). To do that we can simply use the conservation law for \tilde{c}_{-1} which is given in equation (3.41). Note that the only term that contributes is described by the replacement $\tilde{c}_{-1}^{(2)} \rightarrow -\frac{16}{27} \tilde{c}_1^{(2)}$. Taking into account these considerations, we obtain

$$\langle \tilde{c}_1, \tilde{c}_{-1}, \tilde{c}_1 \rangle = -\frac{16}{27} \langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = -\frac{3\sqrt{3}}{4}. \quad (4.10)$$

Following the same procedure, using conservation laws one readily computes some additional terms

$$\langle \tilde{c}_{-1}, \tilde{c}_{-1}, \tilde{c}_1 \rangle = \frac{64}{243} \langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = \frac{1}{\sqrt{3}}, \quad (4.11)$$

$$\langle \tilde{c}_{-1}, \tilde{c}_{-1}, \tilde{c}_{-1} \rangle = 0. \quad (4.12)$$

Let us now consider some correlators involving the operators $\hat{\mathcal{B}}$ and $\hat{\mathcal{L}}$. For example, to compute the coefficient in front of the $x_0^2 x_5$ interaction we write

$$\langle \hat{\mathcal{B}} \hat{\mathcal{L}} \tilde{c}_0 \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = \langle V_3 | \hat{\mathcal{B}}^{(1)} \hat{\mathcal{L}}^{(1)} \tilde{c}_0^{(1)} \tilde{c}_1^{(1)} \tilde{c}_1^{(2)} \tilde{c}_1^{(3)} | 0 \rangle_{(3)} \otimes | 0 \rangle_{(2)} \otimes | 0 \rangle_{(1)}. \quad (4.13)$$

Using the conservation laws (3.22), (3.31) and the following set of commutator and anti-commutator relations

$$[\mathcal{B}_0^{(i)}, \hat{\mathcal{L}}^{(j)}] = \delta^{ij} \hat{\mathcal{B}}^{(j)}, \quad [\mathcal{B}_{-1}^{(i)}, \hat{\mathcal{L}}^{(j)}] = 0, \quad (4.14)$$

$$\{\mathcal{B}_0^{(i)}, \tilde{c}_p^{(j)}\} = \delta^{ij} \delta_{p,0}, \quad \{\mathcal{B}_{-1}^{(i)}, \tilde{c}_p^{(j)}\} = \delta^{ij} \delta_{p,1}, \quad (4.15)$$

$$[\mathcal{L}_0^{(i)}, \tilde{c}_p^{(j)}] = -\delta^{ij} p \tilde{c}_p^{(j)}, \quad [\mathcal{L}_{-1}^{(i)}, \tilde{c}_p^{(j)}] = \delta^{ij} (2-p) \tilde{c}_{p-1}^{(j)}, \quad (4.16)$$

from equation (4.13) we obtain

$$\langle \hat{\mathcal{B}} \hat{\mathcal{L}} \tilde{c}_0 \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = -\frac{8}{9} \langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle - \frac{2}{9} \pi \langle \tilde{c}_0 \tilde{c}_1, \tilde{c}_1, \mathbf{1} \rangle + \frac{1}{18} \pi^2 \langle \tilde{c}_0 \tilde{c}_1, \tilde{c}_0, \mathbf{1} \rangle, \quad (4.17)$$

where the $\mathbf{1}$ appearing in (4.17) represents the $SL(2, \mathbb{R})$ invariant vacuum $|0\rangle$ without any insertion. In order to write this last equation (4.17) in terms of the basic correlator (4.3), we employ the ghost conservation law (3.37)

$$\langle \hat{\mathcal{B}} \hat{\mathcal{L}} \tilde{c}_0 \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = \left(-\frac{8}{9} - \frac{8\pi}{27\sqrt{3}} - \frac{8\pi^2}{243} \right) \langle \tilde{c}_1, \tilde{c}_1, \tilde{c}_1 \rangle = -\frac{9\sqrt{3}}{8} - \frac{3\pi}{8} - \frac{\pi^2}{8\sqrt{3}}. \quad (4.18)$$

We see that in this alternative method, the computation of three point correlation functions avoids the necessity of evaluating any triple contour integral, the conservation laws allow the evaluation of all necessary three point functions in terms of the basic correlator (4.3). All the remaining interaction terms, which come from plugging the string field (4.7) into the definition of the cubic interaction term $V_{\text{inter}} = \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle$, are computed similarly. As a result of our computations, we find that the interacting part of

the potential is given by

$$\begin{aligned}
V_{\text{inter}} = & \frac{27\sqrt{3}}{64}x_0^3 - \frac{81\sqrt{3}}{32}x_0^2x_1 + \left(\frac{27\sqrt{3}}{8} + \frac{\sqrt{3}\pi^2}{16}\right)x_1^2x_0 + \left(-\frac{3\sqrt{3}}{4} - \frac{\pi^2}{8\sqrt{3}} + \frac{\pi^3}{108}\right)x_1^3 \\
& + \left(\frac{27\sqrt{3}}{32} + \frac{9\pi}{16}\right)x_0^2x_2 + \left(-\frac{9\sqrt{3}}{4} - \frac{15\pi}{8} + \frac{\pi^2}{8\sqrt{3}}\right)x_0x_1x_2 + \frac{\sqrt{3}\pi^2}{8}x_2^3 - \frac{3\sqrt{3}}{4}x_0^2x_3 \\
& + \left(\frac{9\pi}{8} + \frac{\sqrt{3}\pi^2}{16}\right)x_0x_2^2 + \left(\frac{\pi^2}{8\sqrt{3}} - \frac{3\pi}{4} - \frac{\pi^3}{36}\right)x_1x_2^2 + \left(\frac{3\sqrt{3}}{4} + \frac{3\pi}{4} - \frac{\sqrt{3}\pi^2}{8} + \frac{\pi^3}{54}\right)x_1^2x_2 \\
& + \left(\sqrt{3} + \frac{\pi}{6}\right)x_0x_1x_3 - \frac{2\pi^2}{9\sqrt{3}}x_1^2x_3 + \left(-\sqrt{3} - \frac{\pi}{6}\right)x_0x_2x_3 - \frac{2\pi^2}{9\sqrt{3}}x_1x_2x_3 + \frac{1}{\sqrt{3}}x_0x_3^2 \\
& + \left(\frac{2}{3\sqrt{3}} - \frac{4\pi}{27}\right)x_1x_3^2 + \frac{2}{3\sqrt{3}}x_2x_3^2 + \left(\frac{27\sqrt{3}}{8} - \frac{\sqrt{3}\pi^2}{8}\right)x_4x_0^2 + \left(-\frac{9\sqrt{3}}{2} - \frac{\pi^3}{36}\right)x_0x_1x_4 \\
& + \left(\frac{3\sqrt{3}}{2} + \frac{3\pi}{2} - \frac{5\pi^3}{108}\right)x_0x_2x_4 + \frac{\pi^4}{81\sqrt{3}}x_1x_2x_4 - \frac{\pi^4}{27\sqrt{3}}x_2^2x_4 + \frac{5\pi^2}{9\sqrt{3}}x_0x_3x_4 \\
& + \left(-\frac{2\pi^2}{9\sqrt{3}} + \frac{4\pi^3}{243}\right)x_1x_3x_4 + \left(\frac{8\pi^3}{243} - \frac{2\pi^2}{9\sqrt{3}}\right)x_2x_3x_4 + \left(\frac{8}{9\sqrt{3}} - \frac{32\pi}{81} - \frac{16\pi^2}{81\sqrt{3}}\right)x_3^2x_4 \\
& + \frac{\pi^4}{36\sqrt{3}}x_4^2x_0 + \left(\frac{2\pi^4}{81\sqrt{3}} + \frac{8\pi^5}{2187} - \frac{4\pi^6}{6561\sqrt{3}}\right)x_4^3 + \left(\frac{32\pi^3}{729} - \frac{8\pi^2}{27\sqrt{3}} - \frac{16\pi^4}{729\sqrt{3}}\right)x_4^2x_3 \\
& + \left(-\frac{23\pi^4}{486\sqrt{3}} + \frac{4\pi^5}{2187}\right)x_4^2x_2 + \left(\frac{\pi^4}{54\sqrt{3}} + \frac{\pi^5}{729}\right)x_4^2x_1 + \left(-\frac{9\sqrt{3}}{8} - \frac{3\pi}{8} - \frac{\pi^2}{8\sqrt{3}}\right)x_0^2x_5 \\
& + \left(\frac{3\sqrt{3}}{2} + \frac{3\pi}{4} + \frac{\pi^2}{2\sqrt{3}} + \frac{\pi^3}{108}\right)x_0x_1x_5 - \frac{\pi^4}{81\sqrt{3}}x_1^2x_5 + \left(\frac{\pi^3}{36} - \frac{3\pi}{2} - \frac{\pi^2}{2\sqrt{3}}\right)x_0x_2x_5 \\
& + \frac{\pi^4}{27\sqrt{3}}x_1x_2x_5 + \frac{\pi^2}{9\sqrt{3}}x_0x_3x_5 + \left(-\frac{4\pi^2}{9\sqrt{3}} - \frac{4\pi^3}{243}\right)x_1x_3x_5 + \left(\frac{8}{9\sqrt{3}} - \frac{8\pi}{81}\right)x_3^2x_5 \\
& + \frac{5\pi^4}{162\sqrt{3}}x_0x_4x_5 + \left(-\frac{2\pi^4}{243\sqrt{3}} + \frac{2\pi^5}{2187}\right)x_1x_4x_5 + \left(-\frac{4\pi^4}{81\sqrt{3}} - \frac{4\pi^5}{729}\right)x_2x_4x_5 \\
& + \left(-\frac{32\pi^2}{27\sqrt{3}} + \frac{8\pi^3}{729} - \frac{16\pi^4}{729\sqrt{3}}\right)x_3x_4x_5 + \left(-\frac{2\pi^4}{243\sqrt{3}} - \frac{16\pi^5}{2187} + \frac{4\pi^6}{2187\sqrt{3}}\right)x_4x_5^2 \\
& - \frac{\pi^4}{108\sqrt{3}}x_0x_5^2 + \left(\frac{5\pi^4}{162\sqrt{3}} + \frac{\pi^5}{729}\right)x_1x_5^2 + \left(-\frac{22\pi^4}{243\sqrt{3}} + \frac{2\pi^5}{2187} - \frac{8\pi^6}{6561\sqrt{3}}\right)x_4^2x_5 \\
& + \frac{\pi^4}{18\sqrt{3}}x_2x_5^2 + \left(\frac{2\pi^4}{27\sqrt{3}} + \frac{2\pi^5}{729}\right)x_5^3. \tag{4.19}
\end{aligned}$$

Once we have obtained the kinetic (4.8) and the cubic interaction term (4.19), we can try to find a critical point of the level (2,6) tachyon potential $V = 2\pi^2(V_{\text{kin}} + V_{\text{inter}})$ without using any gauge. Since in the complete theory there is an infinite-dimensional gauge orbit of equivalent locally stable vacua, we would expect undetermined parameters at the critical value of the potential. Nevertheless the breakdown of gauge invariance caused by level truncation, tell us that the critical points of the level-truncated tachyon

potential have only a discrete set of solutions. At the level (2,6), for example, we find a definite critical point. The critical point is found at $x_0 = 0.218563$, $x_1 = -0.009950$, $x_2 = 0.162003$, $x_3 = 0.053121$, $x_4 = 0.027101$, $x_5 = -0.063514$, and gives the value of $V = -1.049082$ which is 4.90% greater than the required vacuum energy. In contrast, in the Schnabl gauge [27], we have obtained $V_{\text{Sch}} = -1.046622$ which is 4.66% greater than the required vacuum energy (with $x_0 = 0.702361$, $x_1 = 0.165917$, $x_2 = 0.165917$, $x_3 = 0.036787$, $x_4 = 0.044922$, $x_5 = 0.089844$).

We could continue performing higher level computations, since these computations follow the same procedures previously shown, at this stage we only want to present our results. At the next level, namely expanding the string field up to level three states

$$\begin{aligned} \Psi = & x_0 \tilde{c}_1 |0\rangle + x_1 \hat{\mathcal{L}} \tilde{c}_1 |0\rangle + x_2 \hat{\mathcal{B}} \tilde{c}_0 \tilde{c}_1 |0\rangle + x_3 \tilde{c}_{-1} |0\rangle + x_4 \hat{\mathcal{L}}^2 \tilde{c}_1 |0\rangle + x_5 \hat{\mathcal{B}} \hat{\mathcal{L}} \tilde{c}_0 \tilde{c}_1 |0\rangle \\ & + x_6 \hat{\mathcal{L}} \tilde{c}_{-1} |0\rangle + x_7 \hat{\mathcal{L}}^3 \tilde{c}_1 |0\rangle + x_8 \hat{\mathcal{B}} \tilde{c}_{-2} \tilde{c}_1 |0\rangle + x_9 \hat{\mathcal{B}} \tilde{c}_{-1} \tilde{c}_0 |0\rangle + x_{10} \hat{\mathcal{B}} \hat{\mathcal{L}}^2 \tilde{c}_0 \tilde{c}_1 |0\rangle, \end{aligned} \quad (4.20)$$

we have found that there are five critical points for the level (3,9) tachyon potential. The first of these critical points is located at $x_0 = 0.320696$, $x_1 = -0.011758$, $x_2 = 0.142364$, $x_3 = 0.095307$, $x_4 = -0.033742$, $x_5 = -0.032490$, $x_6 = -0.048882$, $x_7 = 0.014538$, $x_8 = -0.095482$, $x_9 = -0.016128$, $x_{10} = 0.008261$ and gives the value of $V = -0.999705$ which is 99.97% of the required vacuum energy. The second point is at $x_0 = 0.375516$, $x_1 = 0.016889$, $x_2 = 0.145665$, $x_3 = 0.089377$, $x_4 = -0.024954$, $x_5 = -0.013322$, $x_6 = -0.036865$, $x_7 = 0.010329$, $x_8 = -0.072036$, $x_9 = -0.009889$, $x_{10} = 0.003576$ and gives the value of $V = -0.999533$ which is 99.95% of the required vacuum energy. The third point is at $x_0 = 0.465295$, $x_1 = 0.038118$, $x_2 = 0.124101$, $x_3 = 0.038559$, $x_4 = -0.019113$, $x_5 = 0.015616$, $x_6 = -0.026136$, $x_7 = -0.002337$, $x_8 = -0.066607$, $x_9 = -0.013154$, $x_{10} = -0.008964$ and gives the value of $V = -1.001516$ which is 100.15% of the required vacuum energy. The fourth point is at $x_0 = 0.670369$, $x_1 = 0.189001$, $x_2 = 0.171491$, $x_3 = 0.103166$, $x_4 = 0.023686$, $x_5 = 0.029369$, $x_6 = -0.000273$, $x_7 = 0.001751$, $x_8 = 0.020806$, $x_9 = 0.022997$, $x_{10} = 0.001284$ and gives the value of $V = -0.999674$ which is 99.96% of the required vacuum energy. The last point is at $x_0 = 0.673922$, $x_1 = -0.072591$, $x_2 = -0.095384$, $x_3 = -0.246487$, $x_4 = -0.041749$, $x_5 = 0.160009$, $x_6 = -0.027547$, $x_7 = -0.006767$, $x_8 = -0.300267$, $x_9 = -0.062356$, $x_{10} = -0.044929$ and gives the value of $V = -1.009647$ which is 100.96% of the required vacuum energy.

Up to the level that we have explored with our computations, we have noticed that as the level of truncation is increased, the multiplicity of the candidate solutions continues to grow. While some solutions approach the correct value, others do not, so without some further criterion for selecting solutions, it does not seem possible to isolate a good candidate for the vacuum in the level-truncated, non-gauge-fixed theory. This difficulty in solving the theory without gauge fixing clearly arises from the presence of a continuous family of gauge equivalent vacua in the full theory. A unique solution of the tachyon potential which represents the stable vacuum at each level has the property that it can be determined from the branch of the gauge fixed effective potential connecting the perturbative and nonperturbative vacua, as it was done in [27] for the tachyon potential in the Schnabl gauge.

5 Summary and discussion

For a string field expanded in the sliver \mathcal{L}_0 -basis, we have provided an efficient and alternative method, based on conservation laws, to compute the three point interaction vertex of the open bosonic string field theory action. As we have seen, by some explicit computations, these conservation laws are easy to apply for low level by hand calculations and they can be implemented for high level computer calculations.

As an application of the conservation laws found in this paper, using the \mathcal{L}_0 level truncation scheme, we have computed the open bosonic string field action relevant to the tachyon condensation. We have explored the possibility of determining the locally stable vacuum without gauge fixing. At higher levels we have found that there are many candidates for the stable vacuum, the multiplicity of these solutions is due to the presence of a continuous family of gauge equivalent vacua in the full theory.

A problem that should be analyzed employing the methods developed in this work would be the evaluation of the tachyon potential in the sliver frame for the case of the modified cubic superstring field theory. The shape of the effective potential in this theory was already conjectured by Erler [53]. This issue is very puzzling since for the case of the modified cubic superstring field theory, the tachyon has vanishing expectation value at the local minimum of the effective potential, so the tachyon vacuum sits directly below the perturbative vacuum.

The procedures developed in our work could be applicable to computations in Berkovits superstring field theory as well. The relevant string field theory is non-polynomial [54], but since the theory is based on Witten's associative star product, the methods discussed in this paper would apply with minor modifications. For instance, we should extend the conservation laws for the case of the n -point interaction vertex which involves the gluing of n -strings with $n \geq 2$.

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